DESCRIPTION OF THE PROGRAM

TwoDoFSim_BB

VERSION 1.0

May 2013.

TABLE OF CONTENTS

1. INTRODUCTION

1.1 GENERAL INFORMATION ABOUT PROGRAM

Purpose and Possibility of the Program

Capability:Calculation of projectile trajectory and firing elements based on Euler equations of motion for point mass (two degrees of freedom).

Method:Numerical integration of Euler differential equations of motion with two degrees of freedom of projectile and integration of differential equation of burning the propellant.

Purpose of program is quick calculation of trajectory of unguided projectile including firing elements (firing problem solution).

It can also be used for:

- the determination of rocket motor thrust characteristics (total impulse, burning time) for the design of artillery rockets,
- the determination of rocket motor thrust characteristics (total impulse, burning time, optimal ignition time) for the extension of range of classical artillery projectiles and mortar shells,
- the design of the base-bleed unit for the extension of range of classical artillery projectiles,

Motion of projectile is modeled by four differential equations of the first order. Progression of burning area is described by first order differential equation. Equations are solved numerically by fourth order Runge-Kutta method.

Burning area with burning thickness of base-bleed fuel can be entered through specified separate file, or calculated in the program for common geometry.

Program is written in Matlab. It automatically prints all characteristic functions of the process in time and produces graphs.

Short Description

Program numerically solves (integrates) four differential equations which define flight of projectile for a given initial conditions

$$\frac{dV_K}{dt} = -g\sin\theta \tag{1.1}$$

$$\frac{d\theta}{dt} = -\frac{g}{V_K}\cos\theta \tag{1.2}$$

$$\frac{dx}{dt} = V_K \cos\theta \tag{1.3}$$

$$\frac{dh}{dt} = -V_K \sin\theta \tag{1.4}$$

$$\frac{d\omega_x}{dt} = \frac{L^a}{J_x} \tag{1.5}$$

and differential equation of burning the fuel (propagation of burning area in time)

$$\frac{dy_b}{dt} = r_b \tag{1.6}$$

In the above system of equations independent variable is time t. The five dependent variables are $\boldsymbol{V}_K$ (or simple $\boldsymbol{V}$) – projectile velocity, $\theta(=\gamma)$ – path angle – angle from horizontal line to the velocity vector, x – range, h – height from ground, ω_x – spin rate, L^a – aerodynamic rolling moment, J_x – axial moment of inertia, and y_b – burning thickness of fuel. Burning rate r_b depends on pressure in the chamber p_c, which itself depends on projectile base pressure, and consequently on ambient atmospheric pressure. Pressure in chamber is determined by solving nonlinear equation which defines quasi steady mass flow rate through BB unit opening.

For the meaning of other quantities see lecture supplement and nomenclature in this manual. Initial conditions are defined at launcher muzzle at initial time which is usually taken to be zero:

$$t = 0: \quad V = V_0, \quad \theta = \theta_0, \quad x = x_0 = 0, \quad h = h_0 = 0, \quad y_b = 0.$$

Terminal condition for process in base-bleed unit is defined by maximal burning thickness:

$$y_b = y_{b.\max}: \quad t = t_{y_b \max}.$$

Terminal condition for trajectory is defined by minimum height (impact to ground):

$$h \le 0: \quad t = t_e.$$

For the integration of system of differential equations the fourth order Runge-Kutta solver is used.

Limitations

Main limitations of the program are:

- Trajectory is in vertical plane – derivation and deviation due to disturbances cannot be calculated.
- Earth is flat and stationary (Coriolis force is not taken into account).

– Only one propellant with homogeneous characteristics can be considered.

– Burning law of base-bleed fuel is exponential (no free term).

Standard Conditions

Standard conditions in this software are defined by:

– Standard gravity $g_{0n} = 9.80665\,\mathrm{m/s^2}$

– Air temperature on MSL $T_a = 15℃$

– Atmospheric pressure on MSL $p_a = 1.013\,\mathrm{bar}$

– Temperature of rocket motor propellant $T_{p.n} = 20℃$

– Temperature of base-bleed fuel $T_{p.f} = 20℃$

2. INPUT DATA AND EXECUTION OF THE PROGRAM

2.1 STRUCTURE AND REVIEW OF INPUT DATA

Basically input data are supplied through two input files:

A. Grain geometry data file FileName.mat (default file name is ybAbVbAplp.mat)

B. Projectile data file FileName.m (default file name is PMMdata_.m).

Both of them are Matlab files (generated by Matlab).

The relations with notation in theory is given in the following table.

Table 1 – Grain geometry data file (ybAbVbAplp.mat)

ybAbVbAplp.mat				
No.	**Quantity in file**	**Symbol in theory**	**Unit**	**Description**
1.	yb	y_b	[m]	Burnt or burning thickness of propellant
2.	Ab	A_b	[m^2]	Burning area
3.	BurntVol Vb	V_b	[m^3]	Volume of burnt mass
4.	BurntMass mb	m_b	[kg]	Burnt mass
5.	Ap	A_p	[m^2]	Port area
6.	lp	l_p	[m]	Port perimeter

User can generate that file either by using analytical formulas or by any 3D solid modeler CAD program (SolidWorks, AutoCAD, CATIA ...) and transfer to Matlab and make grain geometry data file. In both cases finally it should be made in Matlab in order to be written in **.mat** format.

Projectile data file (PMMdata_.m)

File contains necessary data to calculate trajectory and process in base-bleed unit.

Table 2 – Projectile data file (PMMdata_.m)

PMMdata_.m				
No.	Quantity in file	Symbol in theory	Unit	Description
1.	`clc;`	–	–	Matlab statement – clears Matlab work space
2.	`close all;`	–	–	Matlab statement – Closes all Matlab opened figures
3.	`Title`	–	–	Identifier of weapon/projectile. It serves as information purposes – not significant for calculation. It is printed in work space.
4.	`dref`	d	[m]	Reference length - calibre
5.	`Sref`	S	[m^2]	Reference area. Usually $S = d^2\pi/4$
6.	`mass0`	m_0	[kg]	Projectile initial mass
7.	`twistd`	φ	[deg]	Barrel rifling twist angle
8.	`Jx0`	m_0	[kgm^2]	Projectile initial moment of inertia
9.	`iet`	i_{et}	[-]	Drag form factor - correction factor for drag coefficient. For the example in this report (Rdz_Fr_155mm) $i_{et} = 1$, because coded drag coefficient is calculated exactly for the projectile considered in the example. For any other shape of projectile, or any other coded drag coefficient drag form coefficient should be determined to achieve range obtained by experiment. See also section "Coding the Aerodynamic Coefficients" in this report and corresponding LS.
10.	`IdentRM`	–	[-]	RM operation On/Off identifier. For `IdentRM=0` operation of rocket motor is off.. For `IdentRM=1` operation of rocket motor is on.
11.	`Itot`	I_T, I_{tot}	[Ns]	Total impulse of RM
12.	`propmass`	m_p	[kg]	Propellant mass of RM
13.	`tburn`	t_b	[s]	Burning time of RM
14.	`tign`	t_{ign}	[s]	Ignition time of RM
15.	`IdentBB`	–	[-]	BB operation On/Off identifier. For `IdentBB=0` operation of rocket motor is off.. For `IdentBB=1` operation of rocket motor is on.
16.	`tignBB`	$t_{i.\mathrm{BB}}$	[s]	Ignition time of BB
17.	`dbase`	d_b	[m]	Base diameter of projectile – base diameter of BB unit
18.	`Sbase`	S_b	[m^2]	Base area, $S_b = d_b{}^2\pi/4$

19.	rhof	ρ_f	[kg/m^3]	Density of BB fuel
20.	Lp	L_p	[m]	Length of BB grain of BB grain
21.	Ns	N_s	[-]	Number of slots of BB grain
22.	Dp	D_p	[m]	Diameter (outer) of grain of BB fuel
23.	Di	D_i	[m]	Inner diameter of BB grain
24.	bs	b_s	[m]	Slot width of BB grain
25.	dm	d_m	[m]	Diameter of cylindrical part with nozzle. This diameter exists when BB unit is around the RM nozzle. For the design case with no RM $d_m = 0$.
26.	Ae	A_e	m^2	Opening (exit) area
27.	Geom_File()	–	–	Name of file which generates file ybAbVbAplp.mat which contains geometrical characteristics of BB grain and burning geometry with respect to burning thickness. In this example file name is Geom_InternalBurningTubeWithSlots (); See Grain Geometry Data File section. The file can be disabled by inserting character % before the file name. Than file ybAbVbAplp.mat is not generated by running the file, but it may be generated by earlier run of this file, or it can be generated upon the running any other program.
28.	Rgas	R	[J/kgK]	Gas (specific) constant of BB burning product. In this program it is assumed that it is independent of pressure in the chamber.
29.	Tflame	T_f	[K]	Adiabatic flame temperature of BB fuel. In this program it is assumed that it is independent of pressure in the chamber.
30.	kappa	κ	–	Ratio of specific heats. In this program it is assumed that it is independent of pressure in the chamber.
31.	etath	η_θ	–	Coefficient of thermal reduction. It is assumed that mean temperature in chamber is less than flame temperature by amount defined by this coefficient, so that $T_c = \eta_\theta T_f$. Practical values for the coefficient are $\eta_\theta = 0.985 \div 0.995$. The coefficient is not implemented in this version – it will be implemented in the next version.
32.	cetad	ζ_d	–	Coefficient of discharge efficiency. It is assumed that discharge mass flow rate is less that theoretical one by amount defined by this coefficient, so that $\dot{m}_e = \zeta_d \dot{m}_{e.theor}$. Practical values for the coefficient are $\eta_\theta = 0.980 \div 0.995$. The coefficient is not implemented in this version – it will be implemented in the next version.
33.	brb	b	[m/s]	Coefficient in burning rate law defined by $r_b = b p_c^{\ n}$ for standard BB fuel temperature $T_{p.f} = 21^\circ \mathrm{C}$ and stationary BB unit (non-accelerated, non-spinning propellant). Note that value of coefficient is for pressure is in Pascals.
34.	nrb	n	–	Exponent in burning rate law defined by $r_b = b p_c^{\ n}$. The value ranges from 0 to 1 depending of propellant composition. Note that higher values than 1 generate unstable burning process in chamber.
35.	krp	k_1	–	Coefficient of the influence of spin rate on burning rate $r_b = k_1 b p_c^{\ n}$ of BB fuel. It depends on spin rate and of fuel composition. The value ranges from 1.1 to 1.2 for typical spin stabilized projectiles,

				but user can enter experimentally determined value for corresponding spin rate. Note that spin vs. time is calculated in the program and can be used for coding function $k_1 = f(\omega_x)$, ω_x being spin rate (pspin).
36.	betarb	β	[1/K]	Coefficient of the influence of propellant temperature on burning rate $r_b = e^{\beta(T_p - T_{p.n})} k_1 b p_c{}^n$. The value ranges from 0.0012 to 0.0016 depending of propellant composition. The coefficient is not implemented in this version – it will be implemented in the next version.
37.	Ta0c	T_{a0}	[°C]	Ground air temperature
38.	pa0	p_{a0}	[mbar]	Ground air pressure
39.	Windx	W_x	[m/s]	Ground longitudinal wind. Positive value is in the direction of flight (tail wind). Negative value is in opposite direction of flight (head wind).
40.	V0	V_0	[m/s]	Muzzle velocity - Projectile initial velocity with respect to ground. It should be greater than zero.
41.	iTheta0d	θ_{0i}	[deg]	Initial elevation angle, $\theta_{0i} > 0$
42.	dTheta0d	$d\theta_0$	[deg]	Increment of elevation angle
43.	NTheta0d	n_{θ_0}	[-]	Number of elevation angles
44.	Ts	T_s	[s]	Integration step for solving system of differential equations which define process in RM. To large integration step can produce instability in integration. It is recommended to choose 100ms at start and carefully increase the value preserving desired accuracy
45.	jprint	–	–	Printing step ratio – ratio of the printing step and integration step. It is always greater than unity. For example, to print each tenth point obtain by integration, put `jprint=10`
46.	PrintToFileID	–	–	Output file Identifier. `PrintToFileID=0` – Results printed to Matlab Workspace; `PrintToFileID=1` – Results printed to file PPMResults.txt;

2.2 MODELING INPUT DATA FOR ARTILLERY ROCKETS

Total Impulse and Burning Time

Program does not calculate motion of projectile in barrel or on the launcher. The initial velocity should be greater than zero. For artillery rocket user ought to enter muzzle velocity as for the artillery shells. For this reason total impulse of the rocket motor for the remaining of flight should be reduce for the amount consumed to boost rocket to muzzle velocity.

Aerodynamic Drag Coefficient

Aerodynamic drag coefficient is coded in the program (see section "Coding the Aerodynamic

Coefficients”). For spin stabilized projectiles it is suggested to use $C_{D.43}$ etalon coefficient. For fin stabilized projectiles $C_{D.58}$ etalon coefficient is more appropriate (see LS “Basis of External Ballistics - Point Mass Model”). Based on used chosen drag coefficient, drag form factor is to be determined from firing, or from firing tables.

2.1 EXECUTION OF THE PROGRAMS

Execution of the Main program TwoDoFsim_BB.m

To run the main program double click on TwoDoFsim_BB.m. The main program window appears. Or, alternatively run Matlab program first and open the file TwoDoFsim_BB.m. In both cases Matlab environment should be opened.

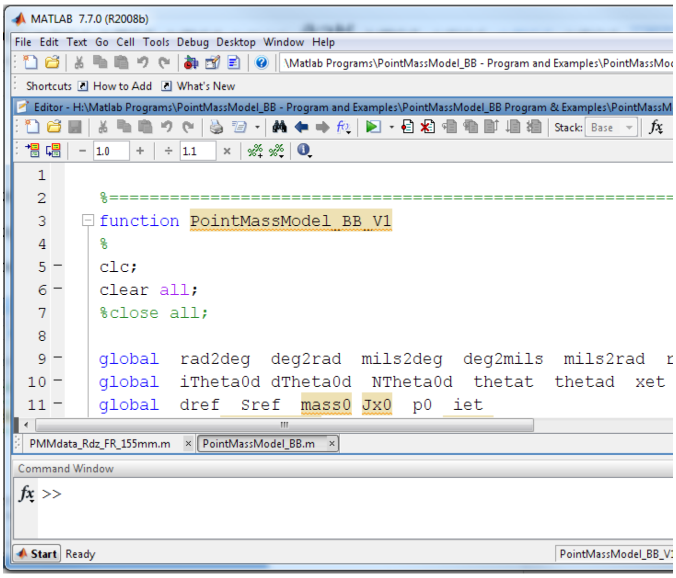


Figure – Matlab environment and main program.

Upon the running of the program the following dialog appears for opening/running file with projectile data,

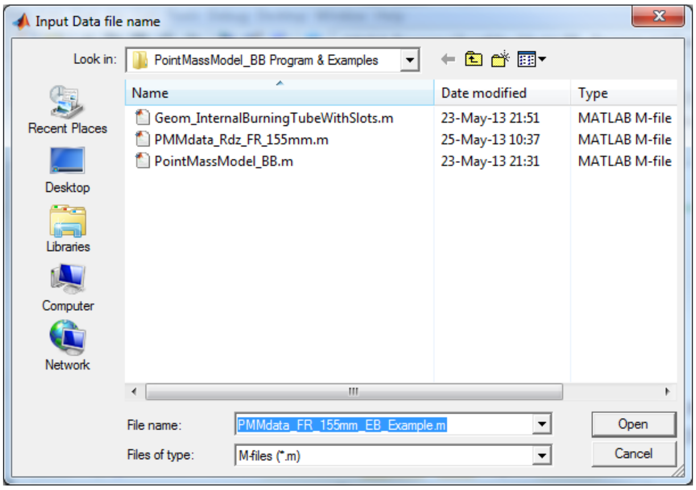


Figure – Dialog for opening file with rocket motor data.

And, after that, dialog for opening/running file with grain geometry data appears.

User can search any directory and choose any appropriate data. Note that default offered file ybAbVbAplp.mat is generated by program `Geom_File` (`Geom_InternalBurningTubeWithSlots.m` in this example) called in `PMMdata_.m` file (`PMMdata_Rdz_FR_155mm.m` in this example).

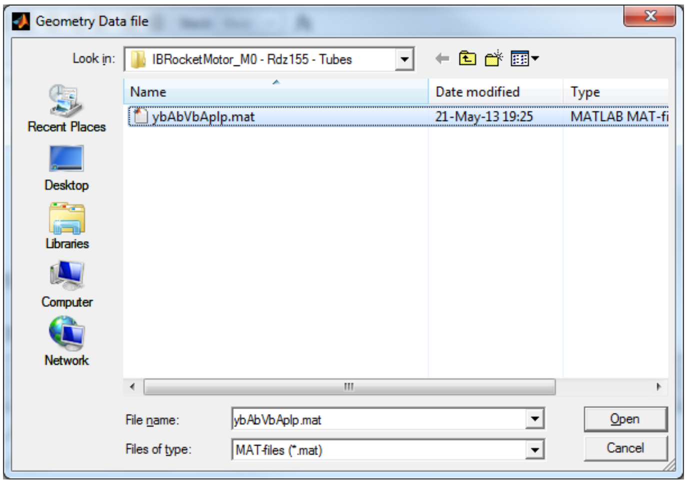


Figure – Dialog for opening file with grain geometry data.

Execution of the Program `Geom_InternalBurningTubeWithSlots.m`

The Program (function)

```
Geom_InternalBurningTubeWithSlots(Lp,Ns,Dp,Di,bs,rhop,PrintID,PlotID);
```

which is used for generating the geometry data, can be called on two ways: with arguments (like above) and without arguments – geometrical parameters. When it is called with arguments it uses arguments which are passed to function. When it is called without arguments it uses geometrical parameters coded in the body of function. For example calling by statement (no arguments)

```
Geom_InternalBurningTubeWithSlots();
```

effects in using the parameters coded in function body

```
%============================================================================
function Geom_InternalBurningTubeWithSlots(Lp,Ns,Dp,Di,bs,rhop,PrintID,PlotID);
% calculation of burning area for internal-Burning Tube with slots
% Side surfaces and outer surface are inhibited.

% Note:  Dimensions in [m]

clc;
%clear all;
%close all;
%------------------------------------------------------------------------
if nargin == 0    % If number of input arguments is zero use the following
                  % values for grain characteristics
   Lp      = 100.0e-3;         % m      Length
   Ns      = 3;                % -      Number of slots
   Dp      = 116.7e-3;         % m      Outer diameter of the grain
   Di      = 60.0e-3;          % m      Inner diameter of the grain
   bs      = 3.0e-3;           % m      Slot width
   web     = 0.5 * (Dp - Di);  % m      Web
   rhop    = 1598;             % kg/m³  Density
   PrintID  = 1;               % Printing identifier;
                               %     PrintID=0 - No printing,
                               %     PrintID=1 - Printed in work space
   PlotID   = 1;               % Ploting identifier;
                               %     PlotID=0 - No ploting,
                               %     PlotID=1 - Graphs are plotted
end;
%------------------------------------------------------------------------
%
…
```

4. RESULTS

4.1 NUMERICAL RESULTS

Upon the successful run of the program results are printed in workspace. Here is an example of eight trajectories (iTheta0d=42.5, NTheta0d=8, dTheta0d=2.5) for one hypothetical 155mm projectile with rocket motor and base-bleed unit. Maximal range is for elevation 56.1deg.

```
TRAJECTORIES FOR SEARCHING MAXIMAL RANGE
    Theta0    Range      El      TOF    Vertex   Vterm  FallAng
    -------  -------  -------  ------  -------  ------  ------
    [deg]      [m]    [mils]     [s]     [m]    [m/s]   [deg]
    -------  -------  -------  -------  ------  ------  ------
    42.5      43873      756    106.5   12070     340     -63
    45.0      45718      800    112.3   13425     346     -64
    47.5      47399      844    117.9   14840     356     -64
    50.0      48792      889    123.4   16307     367     -65
    52.5      49776      933    128.7   17811     379     -66
    55.0      50277      978    133.8   19337     390     -66
    57.5      50246     1022    138.7   20868     401     -67
    60.0      49631     1067    143.3   22388     412     -68
Data at maximal range
    56.1      50342      997    136.0   20013     395     -67
--------------------------------------------------------------------
```

After the calculation of these set of trajectories program calculates and prints results for maximal range.

Table 3 – Data printed in Work space and in Graphs

IBRMResults.txt				
No.	**Quantity in file**	**Symbol in theory**	**Unit**	**Description**
				Standard set of data – printed always
1.	`Theta0`	θ_0	[deg]	Initial elevation
2.	`Range`	x_e	[m]	Range
3.	`El`	QE	[mil]	Initial elevation in mils – Quadrant elevation, (division 6400)
4.	`TOF`	t_e	[s]	Time of flight
5.	`Vertex` `MaxOrd`	h_s	[m]	Maximal ordinate
6.	`Vterm`	V_e	[m/s]	Terminal speed
7.	`FallAng`	θ_e	[deg]	Falling angle
8.	`Time`	t	[s]	Time
9.	`Height`	h	[m]	Height above ground
10.	`VK`	V_K	[m/s]	Velocity with respect to ground
11.	`Theta`	θ_0	[deg]	Elevation
12.	`spin`	ω_x, p	[rad/s]	Spin rate, axial angular speed
13.	`mass`	m	[kg]	Mass
14.	`pa`	p_a	[mbar]	Ambient atmospheric pressure
15.	`pb`	p_b	[mbar]	Base pressure – pressure on the base of projectile
16.	`pc`	p_c	[mbar]	Chamber pressure
17.	`Cred`	C_{red}	[-]	Drag reduction coefficient
18.	`Cpb0`	C_{pb0}	[-]	Base drag coefficient without influence of BB
19.	`Cpb`	C_{pb}	[-]	Base drag coefficient with influence of BB
20.	`Ma`	Ma	[-]	Flight Mach number
21.	`mp, mf`	$m_f(t)$	[kg]	Current mass of BB fuel
22.	`mbdot` `mfdot`	$\dot{m}_b$	[kg/s]	Mass flow rate
23.	`Ab`	A_b	[cm²]	Burning area
24.	`Inject`	I	[-]	Base injection parameter
25.	`Mj`	M_j	[-]	Mach number in BB jet at exit area
26.	`CD0`	C_{D0}	[-]	Drag coefficient without influence of BB
27.	`CD0BB`	$C_{D0.BB}$	[-]	Drag coefficient with influence of BB
28.	`CDb0`	C_{D0}	[-]	Base drag coefficient without influence of BB
29.	`dDrag`	ΔD	[-]	Increment of drag due to influence of BB

Example of the output data is shown below.

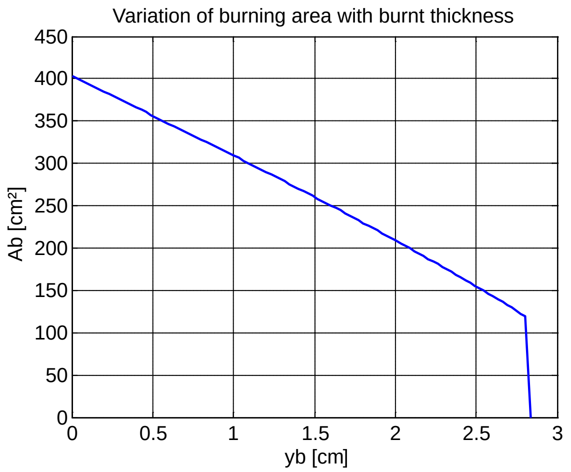
Mass of BB fuel = 1.200 kg
Mass of RM prop. = 3.800 kg

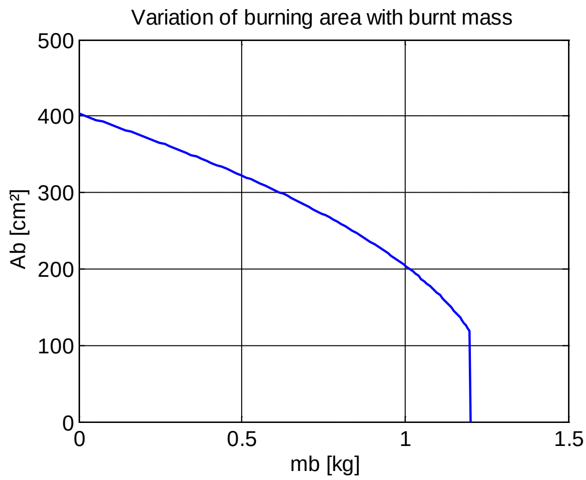
BASIC QUANTITIES OF TRAJECTORY

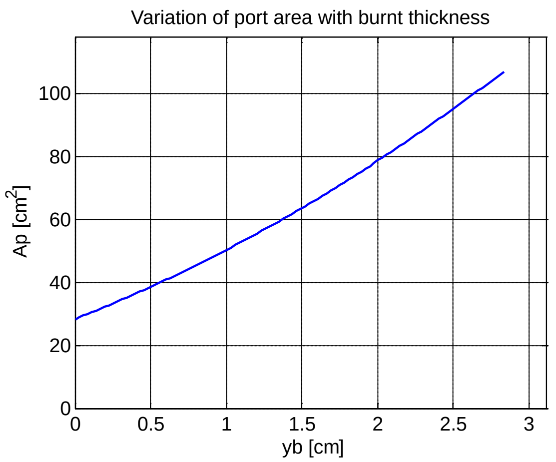
Time	Range	Height	VK	Theta	spin	mass	pa	pb	pc	Cred	Cpb0	Cpb	Ma	mp	mbdot	Ab	Inject	Mj	Time
[s]	[m]	[m]	[m/s]	[deg]	[rad/s]	[kg]	[mbar]	[mbar]	[mbar]	[-]	[-]	[-]	[-]	[kg]	[g/s]	[cm^2]	[10^3]	[-]	[s]
2.000	952.9	1414	813.0	55.6	1771.3	43.60	854	667	733	0.537	-0.115	-0.053	2.427	1.152	48.145	396.3	3.395	0.39	2.00
4.000	1843.0	2697	750.5	54.8	1728.4	43.51	728	584	645	0.559	-0.124	-0.055	2.275	1.060	43.635	382.3	3.793	0.40	4.00
6.000	2686.1	3873	698.6	53.9	1693.3	43.43	626	516	574	0.583	-0.131	-0.055	2.148	0.977	39.818	369.3	4.201	0.41	6.00
8.000	3491.6	4959	654.5	52.9	1664.0	43.35	543	457	512	0.606	-0.138	-0.054	2.040	0.901	36.430	357.0	4.606	0.42	8.00
10.000	4266.5	5965	616.3	51.8	1639.4	43.28	474	407	459	0.627	-0.143	-0.053	1.946	0.831	33.413	345.3	5.008	0.44	10.00
12.000	5016.3	6900	582.7	50.7	1618.1	43.22	416	364	413	0.648	-0.148	-0.052	1.863	0.767	30.719	334.2	5.409	0.45	12.00
14.000	5745.1	7770	552.7	49.4	1599.7	43.16	368	326	373	0.668	-0.153	-0.051	1.788	0.708	28.306	323.6	5.808	0.46	14.00
16.000	6456.2	8580	525.6	48.0	1583.6	43.10	327	294	338	0.686	-0.157	-0.049	1.719	0.653	26.133	313.5	6.203	0.47	16.00
18.000	7169.8	9353	553.5	46.6	1569.4	42.01	292	257	300	0.667	-0.150	-0.050	1.830	0.603	23.875	303.7	5.906	0.49	18.00
20.000	7999.3	10209	638.9	45.3	1556.0	39.88	256	215	255	0.617	-0.132	-0.050	2.139	0.558	21.403	294.6	5.097	0.53	20.00
22.000	8931.6	11130	647.2	44.0	1543.6	39.17	222	185	223	0.617	-0.129	-0.049	2.192	0.517	19.488	286.1	5.167	0.55	22.00
24.000	9853.6	12002	622.4	42.7	1533.0	39.13	193	164	201	0.639	-0.134	-0.048	2.108	0.480	18.010	278.0	5.697	0.57	24.00
26.000	10760.6	12821	600.0	41.4	1524.0	39.09	170	147	183	0.661	-0.138	-0.047	2.033	0.445	16.694	270.2	6.233	0.59	26.00
28.000	11654.9	13589	579.6	39.9	1516.3	39.06	150	132	166	0.681	-0.142	-0.045	1.963	0.413	15.515	262.7	6.769	0.61	28.00
30.000	12538.6	14309	560.8	38.4	1509.5	39.03	134	119	152	0.699	-0.146	-0.044	1.900	0.383	14.456	255.4	7.302	0.63	30.00
32.000	13413.1	14983	543.4	36.8	1503.6	39.00	121	109	140	0.716	-0.150	-0.042	1.841	0.355	13.501	248.5	7.826	0.64	32.00
34.000	14279.6	15612	527.3	35.1	1498.3	38.98	109	99	130	0.731	-0.153	-0.041	1.786	0.329	12.640	241.7	8.337	0.65	34.00
36.000	15139.1	16196	512.3	33.3	1493.5	38.95	100	91	120	0.745	-0.156	-0.040	1.736	0.304	11.869	235.1	8.836	0.67	36.00
38.000	15992.4	16737	498.4	31.4	1489.2	38.93	92	84	112	0.757	-0.160	-0.039	1.688	0.281	11.164	228.7	9.304	0.68	38.00
40.000	16840.3	17236	485.4	29.5	1485.4	38.91	85	79	105	0.766	-0.163	-0.038	1.644	0.259	10.519	222.5	9.737	0.69	40.00
42.000	17683.2	17692	473.4	27.4	1481.8	38.89	79	73	99	0.775	-0.167	-0.038	1.604	0.239	9.928	216.5	10.126	0.69	42.00
44.000	18521.5	18107	462.3	25.2	1478.5	38.87	74	69	93	0.782	-0.170	-0.037	1.566	0.220	9.384	210.5	10.464	0.70	44.00
46.000	19355.8	18481	452.1	23.0	1475.5	38.85	70	65	88	0.788	-0.172	-0.037	1.532	0.201	8.884	204.7	10.744	0.70	46.00
48.000	20186.2	18814	442.8	20.7	1472.7	38.83	66	62	84	0.793	-0.175	-0.036	1.500	0.184	8.423	199.0	10.960	0.69	48.00
50.000	21013.1	19107	434.5	18.3	1470.0	38.82	63	60	80	0.797	-0.177	-0.036	1.472	0.168	7.997	193.4	11.106	0.69	50.00
52.000	21836.7	19359	427.0	15.8	1467.6	38.80	61	57	76	0.800	-0.179	-0.036	1.447	0.152	7.602	187.9	11.177	0.68	52.00
54.000	22657.2	19571	420.6	13.2	1465.2	38.79	59	56	73	0.803	-0.180	-0.035	1.425	0.137	7.236	182.4	11.170	0.67	54.00
56.000	23474.6	19743	415.0	10.6	1463.0	38.77	57	54	71	0.806	-0.182	-0.035	1.406	0.123	6.896	177.0	11.084	0.65	56.00
58.000	24289.1	19875	410.4	7.9	1460.8	38.76	56	53	68	0.807	-0.183	-0.035	1.390	0.110	6.589	171.7	10.934	0.64	58.00
60.000	25100.9	19968	406.8	5.1	1458.7	38.75	55	52	67	0.808	-0.184	-0.035	1.378	0.097	6.303	166.4	10.708	0.62	60.00
62.000	25909.8	20021	404.2	2.4	1456.6	38.73	55	52	65	0.808	-0.185	-0.036	1.369	0.084	6.037	161.2	10.411	0.60	62.00
64.000	26716.1	20035	402.5	-0.4	1454.6	38.72	54	52	64	0.805	-0.185	-0.036	1.363	0.073	5.789	156.0	10.046	0.58	64.00
66.000	27519.7	20009	401.8	-3.2	1452.6	38.71	55	52	63	0.801	-0.186	-0.037	1.361	0.061	5.555	150.7	9.619	0.55	66.00
68.000	28320.6	19945	402.0	-6.0	1450.5	38.70	55	53	63	0.795	-0.185	-0.038	1.362	0.050	5.343	145.5	9.154	0.53	68.00
70.000	29118.8	19841	403.1	-8.8	1448.5	38.69	56	53	63	0.787	-0.185	-0.039	1.366	0.040	5.149	140.2	8.655	0.51	70.00
72.000	29914.1	19699	405.1	-11.5	1446.4	38.68	57	54	63	0.776	-0.184	-0.041	1.372	0.030	4.964	134.9	8.119	0.48	72.00
74.000	30706.6	19518	408.0	-14.2	1444.2	38.67	59	56	63	0.762	-0.184	-0.044	1.382	0.020	4.788	129.6	7.557	0.45	74.00
76.000	31496.1	19299	411.6	-16.8	1441.9	38.66	61	57	64	0.745	-0.182	-0.046	1.394	0.011	4.617	124.1	6.976	0.42	76.00
78.000	32282.5	19041	416.0	-19.4	1439.6	38.65	64	58	62	0.643	-0.181	-0.065	1.409	0.002	2.899	79.4	4.162	0.33	78.00
80.000	33065.0	18746	420.5	-21.9	1437.1	38.65	67	50	51	0.000	-0.180	-0.180	1.425	0.000	0.134	4.1	0.181	0.00	80.00
82.000	33842.4	18413	425.2	-24.4	1434.4	38.65	70	52	52	0.000	-0.179	-0.179	1.440	0.000	0.000	3.8	0.000	0.00	82.00
84.000	34614.0	18044	430.4	-26.7	1431.6	38.65	75	55	55	0.000	-0.178	-0.178	1.458	0.000	0.000	3.8	0.000	0.00	84.00
86.000	35379.6	17639	436.0	-29.0	1428.6	38.65	79	58	58	0.000	-0.177	-0.177	1.477	0.000	0.000	3.8	0.000	0.00	86.00
88.000	36138.8	17198	442.1	-31.3	1425.3	38.65	85	62	62	0.000	-0.175	-0.175	1.498	0.000	0.000	3.8	0.000	0.00	88.00
90.000	36891.1	16722	448.3	-33.4	1421.8	38.65	92	66	66	0.000	-0.173	-0.173	1.519	0.000	0.000	3.8	0.000	0.00	90.00
92.000	37636.0	16211	454.8	-35.4	1417.9	38.65	100	71	71	0.000	-0.172	-0.172	1.541	0.000	0.000	3.8	0.000	0.00	92.00
94.000	38373.0	15667	461.3	-37.4	1413.7	38.65	108	77	77	0.000	-0.170	-0.170	1.563	0.000	0.000	3.8	0.000	0.00	94.00
96.000	39101.3	15091	467.7	-39.3	1409.1	38.65	119	84	83	0.000	-0.168	-0.168	1.584	0.000	0.000	3.8	0.000	0.00	96.00
98.000	39820.2	14482	473.9	-41.1	1404.0	38.65	131	91	91	0.000	-0.167	-0.167	1.606	0.000	0.000	3.8	0.000	0.00	98.00
100.000	40528.8	13844	479.8	-42.9	1398.3	38.65	145	100	100	0.000	-0.165	-0.165	1.626	0.000	0.000	3.8	0.000	0.00	100.00
102.000	41226.2	13177	485.2	-44.6	1392.1	38.65	161	111	111	0.000	-0.163	-0.163	1.644	0.000	0.000	3.8	0.000	0.00	102.00
104.000	41911.3	12483	490.0	-46.2	1385.1	38.65	179	123	123	0.000	-0.162	-0.162	1.660	0.000	0.000	3.8	0.000	0.00	104.00
106.000	42582.8	11763	493.9	-47.7	1377.4	38.65	201	137	137	0.000	-0.161	-0.161	1.673	0.000	0.000	3.8	0.000	0.00	106.00
108.000	43239.5	11021	496.8	-49.2	1368.7	38.65	226	154	153	0.000	-0.160	-0.160	1.683	0.000	0.000	3.8	0.000	0.00	108.00
110.000	43879.7	10259	498.6	-50.7	1359.0	38.65	254	174	173	0.000	-0.161	-0.161	1.671	0.000	0.000	3.8	0.000	0.00	110.00
112.000	44502.3	9479	499.2	-52.1	1348.3	38.65	286	197	196	0.000	-0.163	-0.163	1.654	0.000	0.000	3.8	0.000	0.00	112.00
114.000	45105.7	8684	498.4	-53.5	1336.7	38.65	322	223	223	0.000	-0.164	-0.164	1.633	0.000	0.000	3.8	0.000	0.00	114.00
116.000	45688.7	7878	496.3	-54.8	1323.9	38.65	362	253	252	0.000	-0.166	-0.166	1.608	0.000	0.000	3.8	0.000	0.00	116.00
118.000	46249.9	7064	492.6	-56.1	1309.9	38.65	407	287	286	0.000	-0.169	-0.169	1.578	0.000	0.000	3.8	0.000	0.00	118.00
120.000	46788.0	6244	487.4	-57.3	1294.7	38.65	456	326	324	0.000	-0.171	-0.171	1.545	0.000	0.000	3.8	0.000	0.00	120.00
122.000	47301.9	5423	480.7	-58.6	1278.3	38.65	510	369	368	0.000	-0.174	-0.174	1.507	0.000	0.000	3.8	0.000	0.00	122.00
124.000	47790.4	4605	472.4	-59.8	1260.7	38.65	569	417	416	0.000	-0.177	-0.177	1.466	0.000	0.000	3.8	0.000	0.00	124.00
126.000	48252.8	3792	462.6	-61.0	1242.2	38.65	633	472	470	0.000	-0.181	-0.181	1.421	0.000	0.000	3.8	0.000	0.00	126.00
128.000	48688.4	2988	451.4	-62.1	1222.7	38.65	702	531	530	0.000	-0.184	-0.184	1.373	0.000	0.000	3.8	0.000	0.00	128.00
130.000	49096.6	2197	438.9	-63.3	1202.5	38.65	776	596	594	0.000	-0.189	-0.189	1.323	0.000	0.000	3.8	0.000	0.00	130.00
132.000	49477.5	1421	425.4	-64.4	1181.4	38.65	854	667	665	0.000	-0.194	-0.194	1.270	0.000	0.000	3.8	0.000	0.00	132.00
134.000	49831.0	663	411.0	-65.6	1159.1	38.65	936	743	741	0.000	-0.199	-0.199	1.216	0.000	0.000	3.8	0.000	0.00	134.00
135.800	50126.1	-2	397.4	-66.6	1138.0	38.65	1013	817	814	0.000	-0.204	-0.204	1.167	0.000	0.000	3.8	0.000	0.00	135.80

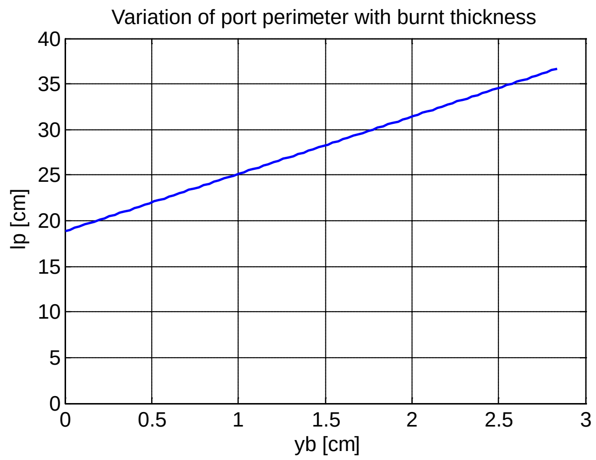
4.2 GRAPHS – DIAGRAMS

Graphs Generated by Grain Geometry Data File



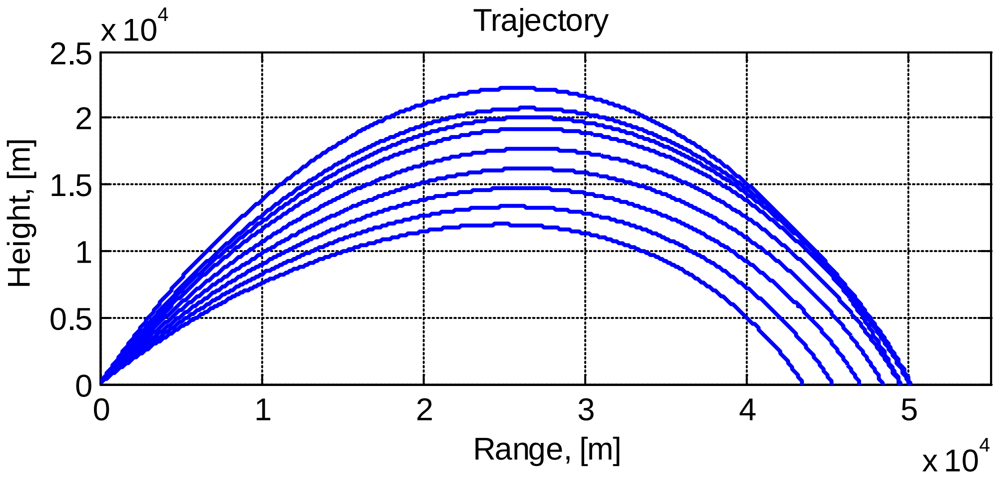


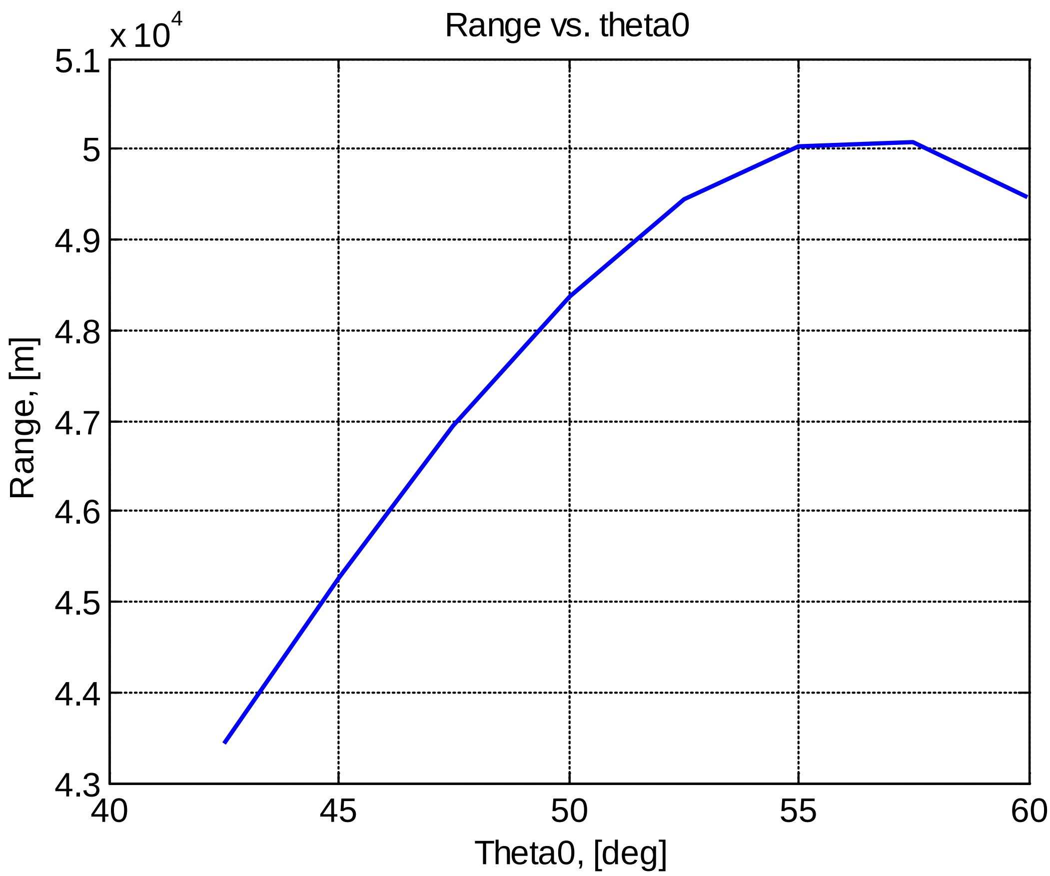




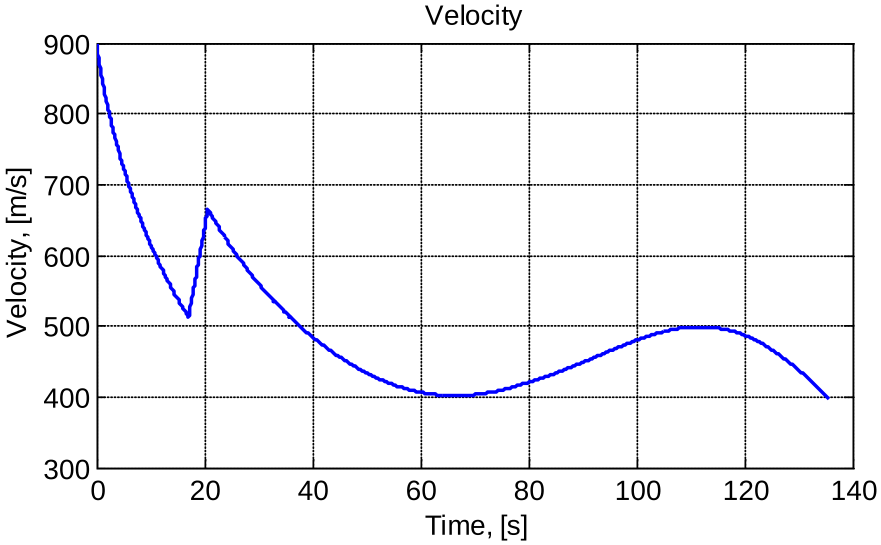
Graphs Generated by Main program

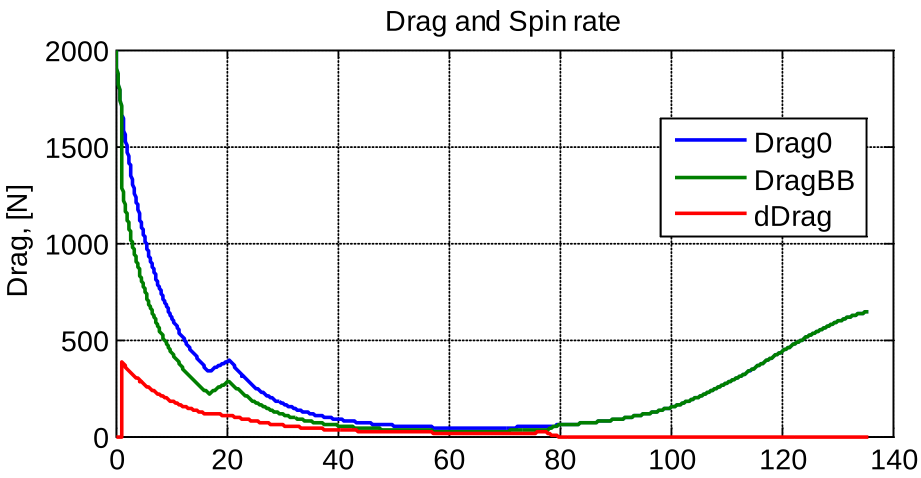
Trajectories for Searching Maximal Range

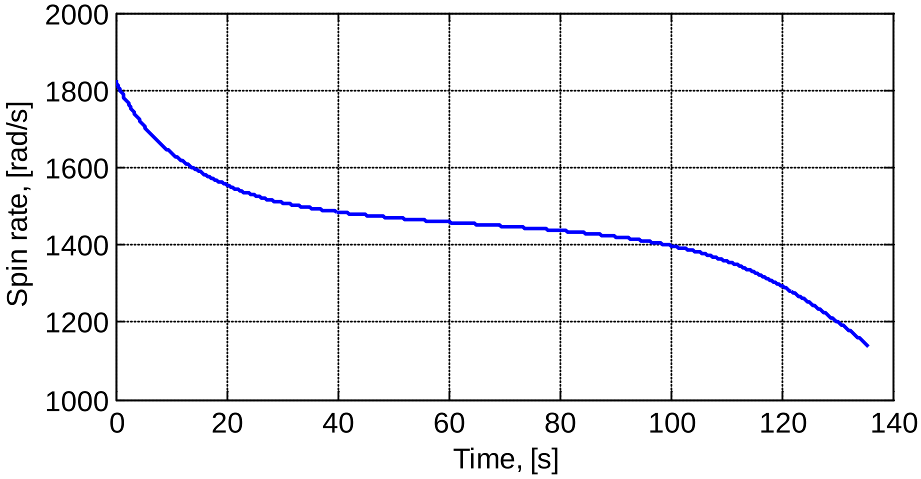


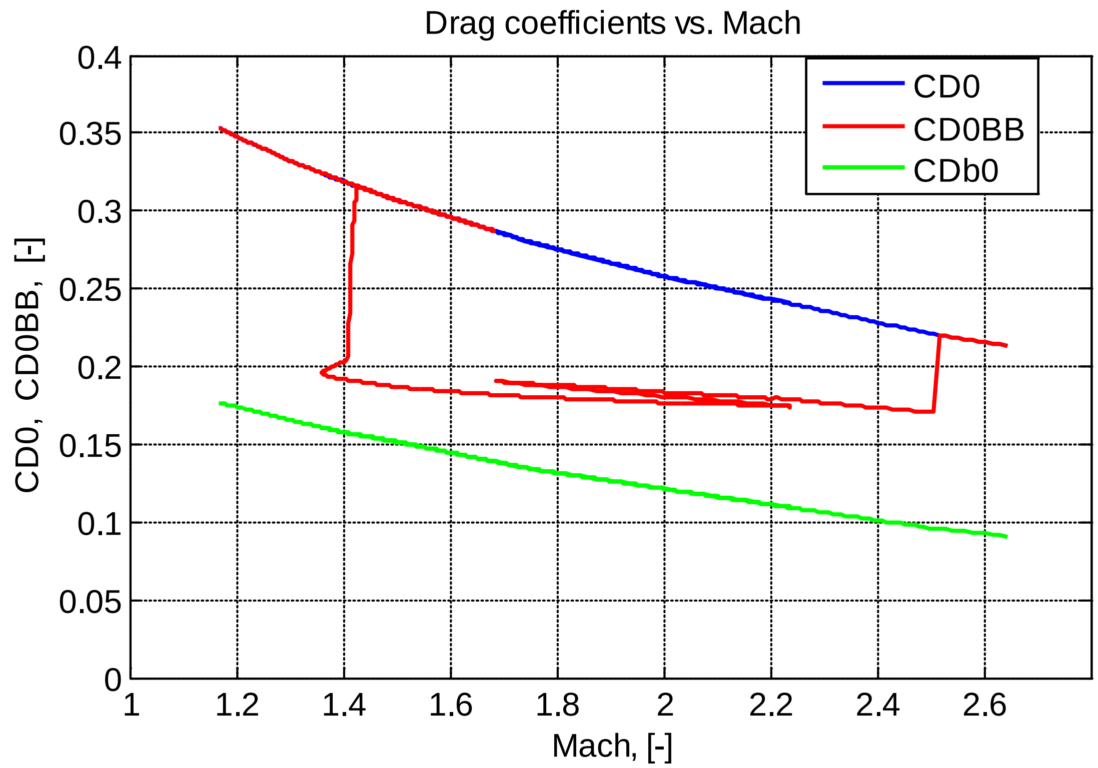


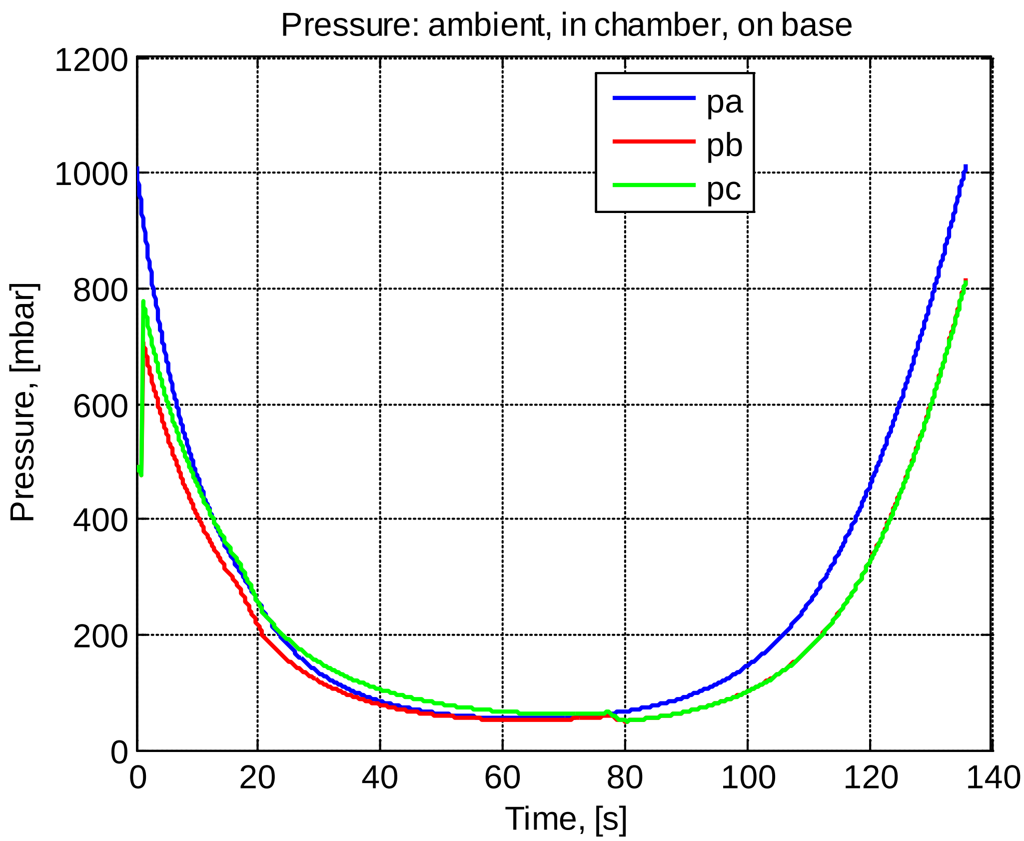
Quantities on Trajectory with Maximal Range

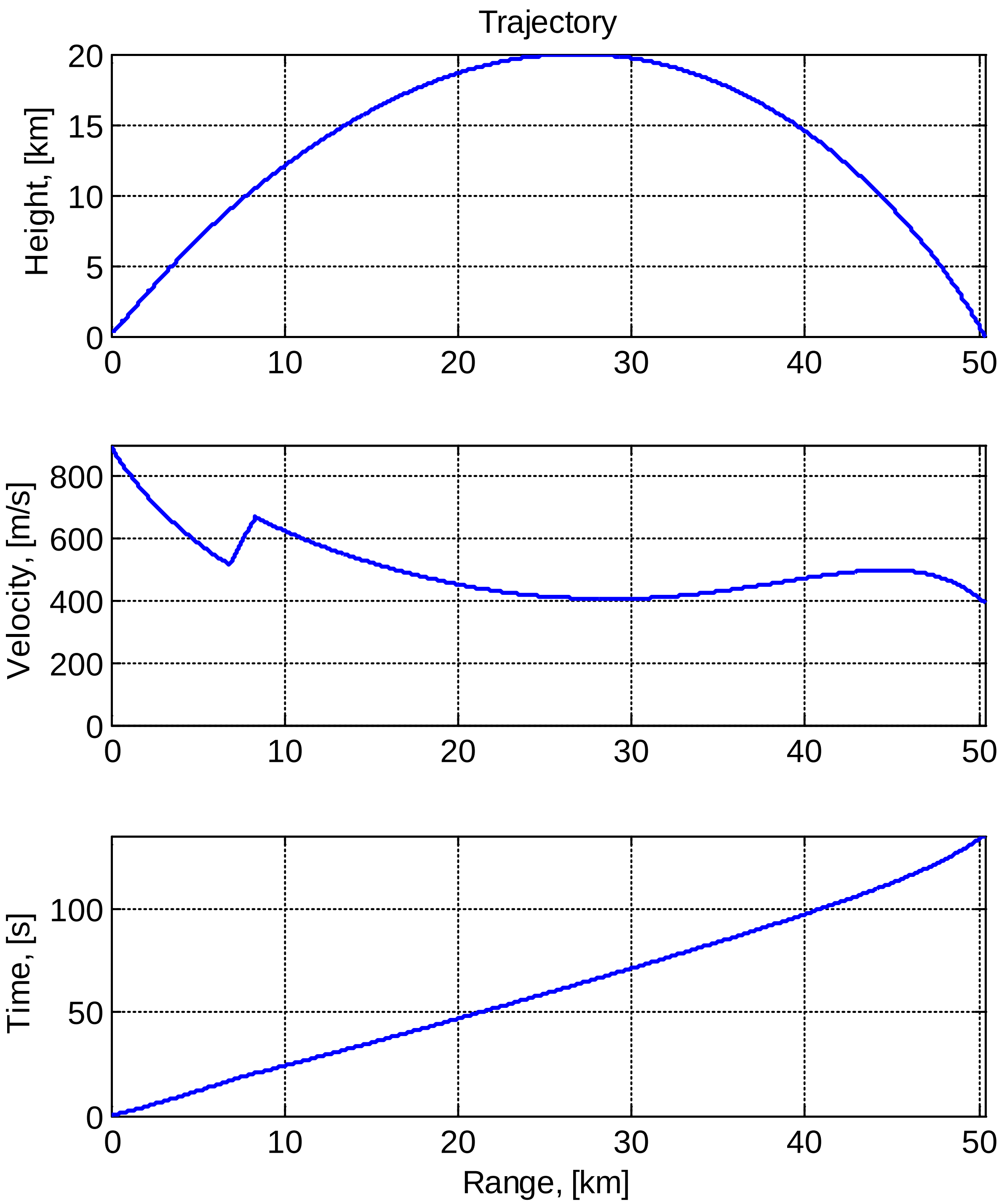


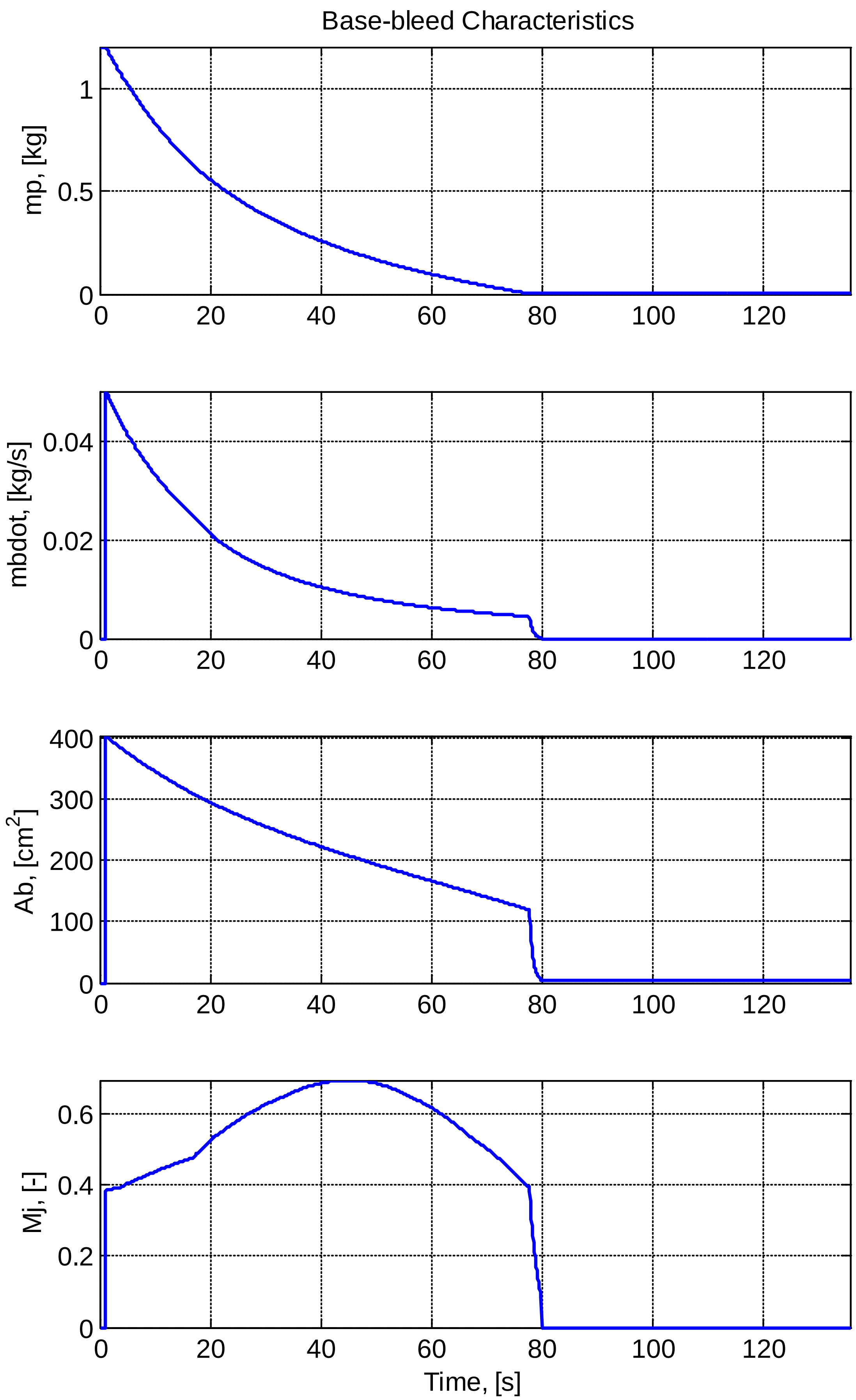


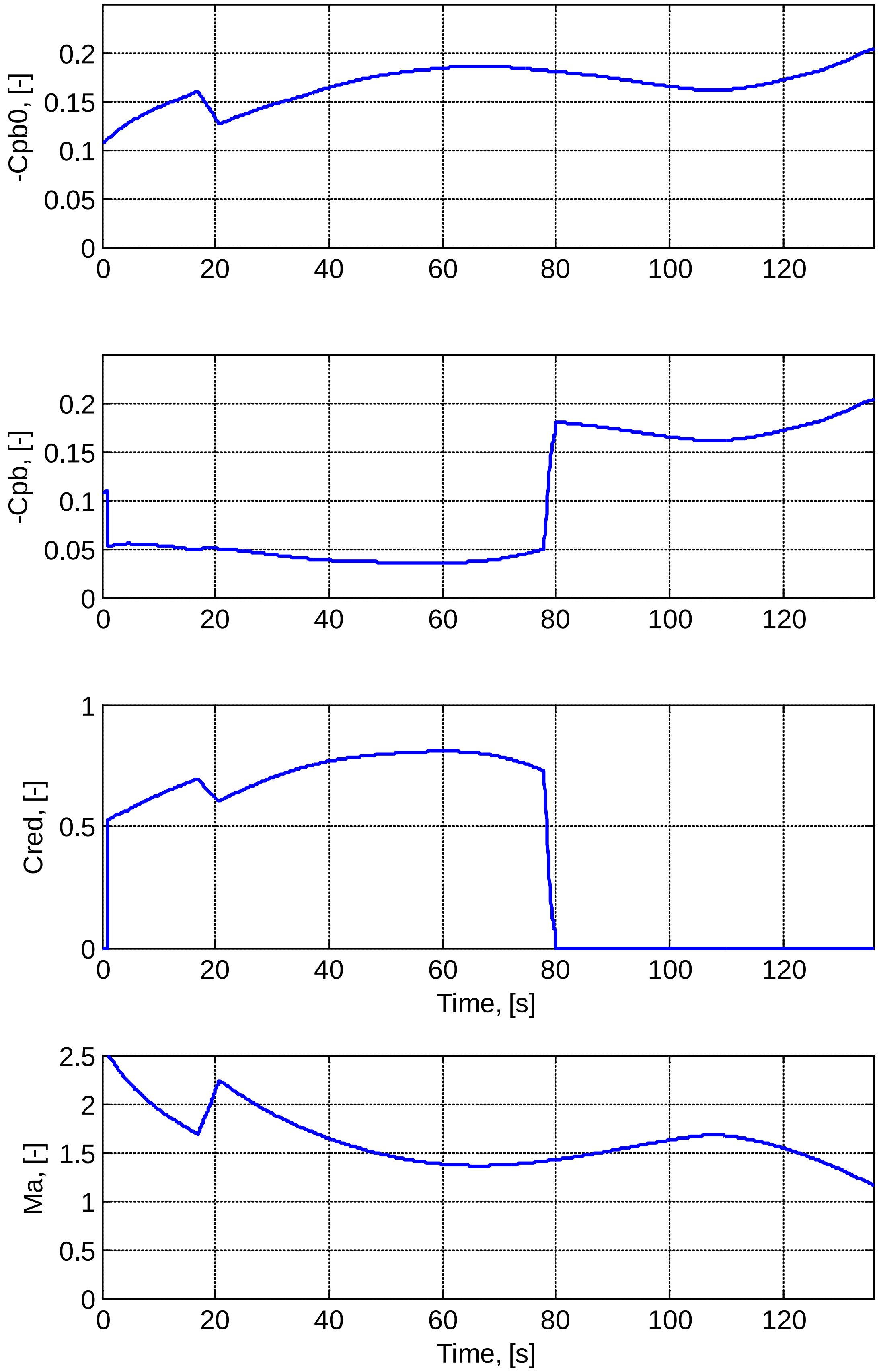



Drag coefficients vs. Mach
CD0
CD0BB
CDb0
CD0, CD0BB, [-]
Mach, [-]
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6


Pressure: ambient, in chamber, on base
pa
pb
pc
Pressure, [mbar]
Time, [s]
1200
1000
800
600
400
200
0
20
40
60
80
100
120
140


Trajectory
Height, [km]
0
5
10
15
20
0
10
20
30
40
50
Velocity, [m/s]
0
200
400
600
800
0
10
20
30
40
50
Time, [s]
0
50
100
0
10
20
30
40
50
Range, [km]


Base-bleed Characteristics
mp, [kg]
1
0.5
0
mbdot, [kg/s]
0.04
0.02
0
Ab, [cm2]
400
300
200
100
0
Mj, [-]
0.6
0.4
0.2
0
0 20 40 60 80 100 120
Time, [s]


-Cpb0, [-]
-Cpb, [-]
Cred, [-]
Time, [s]
Ma, [-]
Time, [s]